**Module 5 - Probability**

**Basics of Probability**

1. Write a Python program to simulate the following scenarios:

  a. Tossing a coin 10,000 times and calculating the experimental probability of heads and tails.

  b. Rolling two dice and computing the probability of getting a sum of 7.

**Steps**

      a. Use Python's random module for simulations.

      b. Implement loops for repeated trials.

      c. Track outcomes and compute probabilities.

2. Write a function to estimate the probability of getting at least one "6" in 10 rolls of a fair die.

**Steps**

      a. Simulate rolling a die 10 times using a loop.

      b. Track trials where at least one "6" occurs.

      c. Calculate the proportion of successful trials.

**Conditional Probability and Bayes' Theorem**

3. A bag contains 5 red, 7 green, and 8 blue balls. A ball is drawn randomly, its color noted, and it is put back into the bag. If this process is repeated 1000 times, write a Python program to estimate:

  a. The probability of drawing a red ball given that the previous ball was blue.

  b. Verify Bayes' theorem with the simulation results.

**Steps**

    a. Use random sampling to simulate the process.

    b. Compute conditional probabilities directly from the data.

**Random Variables and Discrete Probability**

4. Generate a sample of size 1000 from a discrete random variable with the following distribution:

  - P(X=1) = 0.25

  - P(X=2) = 0.35

  - P(X=3) = 0.4

  Compute the empirical mean, variance, and standard deviation of the sample.

  Steps

      a. Use numpy.random.choice() to generate the sample.

      b. Use numpy methods to calculate mean, variance, and standard deviation.

**Continuous Random Variables**

5. Simulate 2000 random samples from an exponential distribution with a mean of 5. Visualize the distribution using:

  a. A histogram.

  b. A probability density function (PDF) overlay.

  Steps

      a. Use numpy.random.exponential().

      b. Use matplotlib to create visualizations.

**Central Limit Theorem**

6. Simulate the Central Limit Theorem by following these steps

  a. Generate 10,000 random numbers from a uniform distribution.

  b. Draw 1000 samples of size n = 30.

  c. Calculate and visualize the distribution of sample means.

  Steps

      a. Use numpy.random.uniform().

      b. Plot both the uniform distribution and the sample mean distribution for comparison.